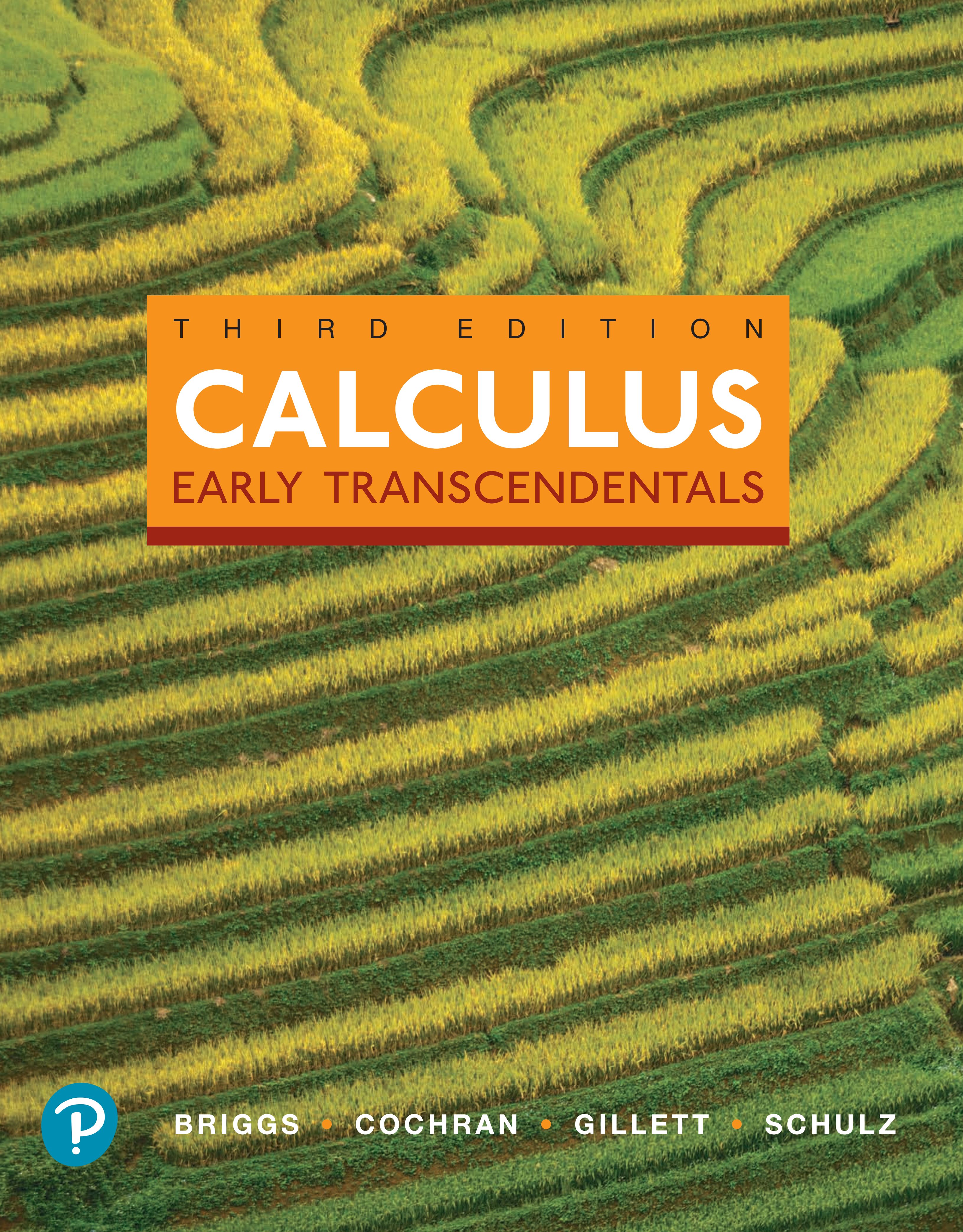

THIRD EDITION

CALCULUS

EARLY TRANSCENDENTALS

BRIGGS • COCHRAN • GILLETT • SCHULZ

MyLab Math for *Calculus: Early Transcendentals*, 3e

(access code required)

Pearson MyLab

Used by over 3 million students a year, MyLab™ Math is the world's leading online program for teaching and learning mathematics. MyLab Math for *Calculus: Early Transcendentals,* 3e delivers text-specific assessment, tutorials, and multimedia resources that provide engaging and personalized experiences, so learning can happen in any environment and course format.

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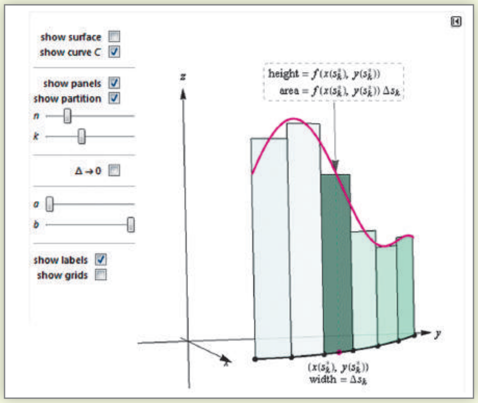

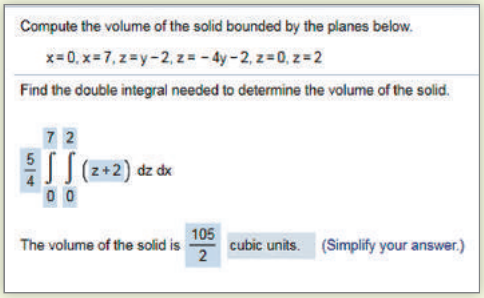

Questions that Deepen Understanding

MyLab Math includes a variety of question types designed to help students succeed in the course. In **Setup & Solve** questions, students show how they set up a problem as well as the solution, better mirroring what is required on tests. **Additional Conceptual Questions** were written by faculty at Cornell University to support deeper, theoretical understanding of the key concepts in calculus.

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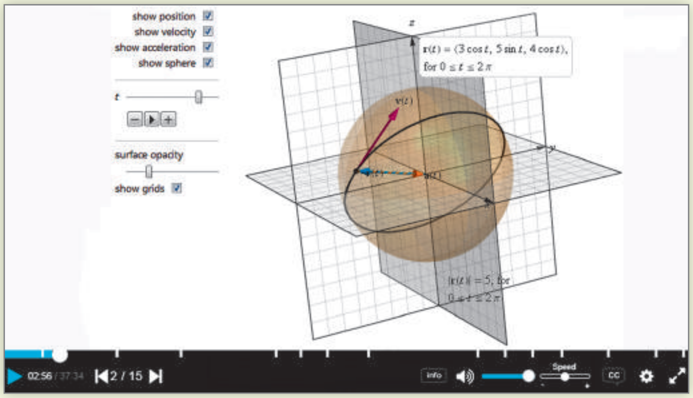


pearson.com/mylab/math

ALGEBRA

Exponents and Radicals

$$x^a x^b = x^{a+b} \qquad \frac{x^a}{x^b} = x^{a-b} \qquad x^{-a} = \frac{1}{x^a} \qquad (x^a)^b = x^{ab} \qquad \left(\frac{x}{y}\right)^a = \frac{x^a}{y^a}$$

$$x^{1/n} = \sqrt[n]{x} \qquad x^{m/n} = \sqrt[n]{x^m} = (\sqrt[n]{x})^m \qquad \sqrt[n]{xy} = \sqrt[n]{x}\sqrt[n]{y} \qquad \sqrt[n]{x/y} = \sqrt[n]{x}/\sqrt[n]{y}$$

Factoring Formulas

$a^2 - b^2 = (a - b)(a + b)$ $\qquad$ $a^2 + b^2$ does not factor over real numbers.

$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$ $\qquad$ $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

$a^n - b^n = (a - b)(a^{n-1} + a^{n-2}b + a^{n-3}b^2 + \cdots + ab^{n-2} + b^{n-1})$

Binomials

$(a \pm b)^2 = a^2 \pm 2ab + b^2$

$(a \pm b)^3 = a^3 \pm 3a^2b + 3ab^2 \pm b^3$

Binomial Theorem

$$(a + b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \cdots + \binom{n}{n-1}ab^{n-1} + b^n,$$

where $\displaystyle\binom{n}{k} = \frac{n(n-1)(n-2)\cdots(n-k+1)}{k(k-1)(k-2)\cdots 3\cdot 2\cdot 1} = \frac{n!}{k!(n-k)!}$

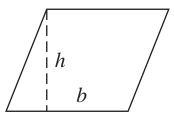
Quadratic Formula

The solutions of $ax^2 + bx + c = 0$ are

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

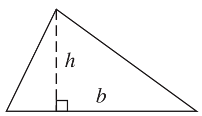
GEOMETRY

Parallelogram



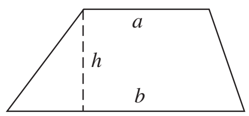
$A = bh$

Triangle



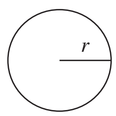
$A = \frac{1}{2}bh$

Trapezoid



$A = \frac{1}{2}(a + b)h$

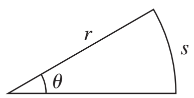
Circle



$A = \pi r^2$

$C = 2\pi r$

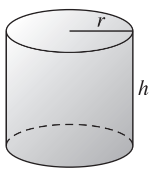
Sector



$A = \frac{1}{2}r^2\theta$

$s = r\theta$ (θ in radians)

Cylinder

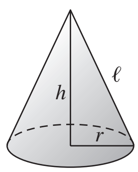


$V = \pi r^2 h$

$S = 2\pi rh$

(lateral surface area)

Cone

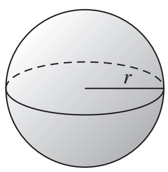


$V = \frac{1}{3}\pi r^2 h$

$S = \pi r\ell$

(lateral surface area)

Sphere

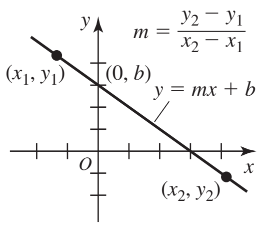


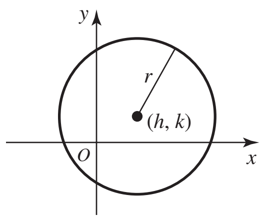
$V = \frac{4}{3}\pi r^3$

$S = 4\pi r^2$

Equations of Lines and Circles

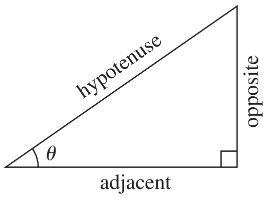
$m = \dfrac{y_2 - y_1}{x_2 - x_1}$	slope of line through (x_1, y_1) and (x_2, y_2)
$y - y_1 = m(x - x_1)$	point–slope form of line through (x_1, y_1) with slope m
$y = mx + b$	slope–intercept form of line with slope m and y-intercept $(0, b)$
$(x - h)^2 + (y - k)^2 = r^2$	circle of radius r with center (h, k)





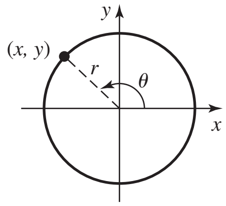
$(x - h)^2 + (y - k)^2 = r^2$

TRIGONOMETRY



$$\cos\theta = \frac{\text{adj}}{\text{hyp}} \qquad \sin\theta = \frac{\text{opp}}{\text{hyp}} \qquad \tan\theta = \frac{\text{opp}}{\text{adj}}$$

$$\sec\theta = \frac{\text{hyp}}{\text{adj}} \qquad \csc\theta = \frac{\text{hyp}}{\text{opp}} \qquad \cot\theta = \frac{\text{adj}}{\text{opp}}$$

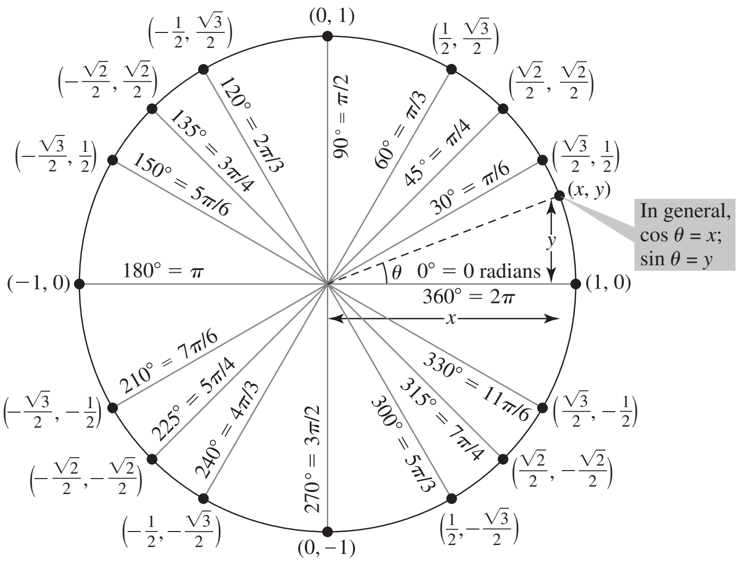


$$\cos\theta = \frac{x}{r} \qquad \sec\theta = \frac{r}{x}$$

$$\sin\theta = \frac{y}{r} \qquad \csc\theta = \frac{r}{y}$$

$$\tan\theta = \frac{y}{x} \qquad \cot\theta = \frac{x}{y}$$

(Continued)



Reciprocal Identities

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \quad \cot\theta = \frac{\cos\theta}{\sin\theta} \quad \sec\theta = \frac{1}{\cos\theta} \quad \csc\theta = \frac{1}{\sin\theta}$$

Pythagorean Identities

$$\sin^2\theta + \cos^2\theta = 1 \quad \tan^2\theta + 1 = \sec^2\theta \quad 1 + \cot^2\theta = \csc^2\theta$$

Sign Identities

$$\sin(-\theta) = -\sin\theta \quad \cos(-\theta) = \cos\theta \quad \tan(-\theta) = -\tan\theta$$
$$\csc(-\theta) = -\csc\theta \quad \sec(-\theta) = \sec\theta \quad \cot(-\theta) = -\cot\theta$$

Double-Angle Identities

$$\sin 2\theta = 2\sin\theta\cos\theta \qquad \cos 2\theta = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$$

$$\tan 2\theta = \frac{2\tan\theta}{1 - \tan^2\theta}$$

Half-Angle Identities

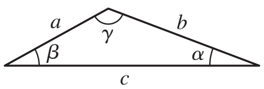
$$\cos^2\theta = \frac{1 + \cos 2\theta}{2} \qquad \sin^2\theta = \frac{1 - \cos 2\theta}{2}$$

Addition Formulas

$$\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta \qquad \sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$$
$$\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta \qquad \cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$$
$$\tan(\alpha + \beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta} \qquad \tan(\alpha - \beta) = \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}$$

Law of Sines

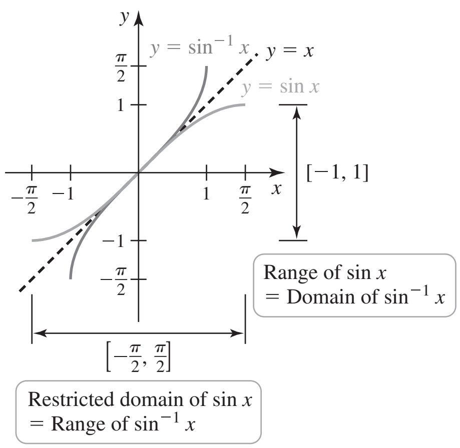
$$\frac{\sin\alpha}{a} = \frac{\sin\beta}{b} = \frac{\sin\gamma}{c}$$

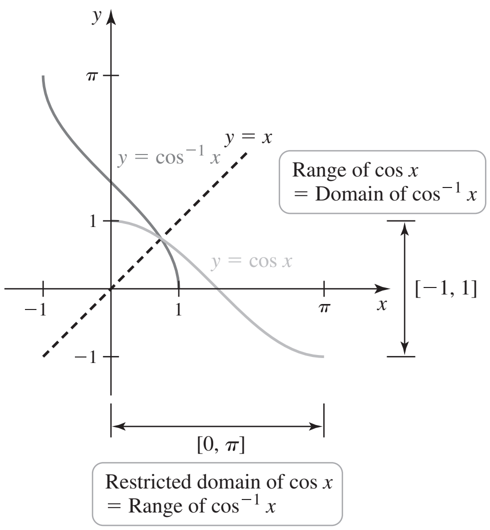


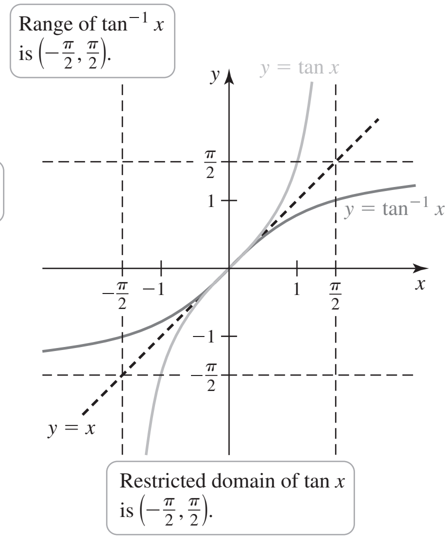
Law of Cosines

$$a^2 = b^2 + c^2 - 2bc\cos\alpha$$

Graphs of Trigonometric Functions and Their Inverses







Calculus

EARLY TRANSCENDENTALS

Third Edition

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For Julie, Susan, Sally, Sue,
Katie, Jeremy, Elise, Mary, Claire, Katie, Chris, and Annie,
whose support, patience, and encouragement made this book possible.

Contents

Preface

The third edition of *Calculus: Early Transcendentals* supports a three-semester or four-quarter calculus sequence typically taken by students studying mathematics, engineering, the natural sciences, or economics. The third edition has the same goals as the first edition:

- to motivate the essential ideas of calculus with a lively narrative, demonstrating the utility of calculus with applications in diverse fields;
- to introduce new topics through concrete examples, applications, and analogies, appealing to students' intuition and geometric instincts to make calculus natural and believable; and
- once this intuitive foundation is established, to present generalizations and abstractions and to treat theoretical matters in a rigorous way.

The third edition both builds on the success of the previous two editions and addresses the feedback we have received. We have listened to and learned from the instructors who used the text. They have given us wise guidance about how to make the third edition an even more effective learning tool for students and a more powerful resource for instructors. Users of the text continue to tell us that it mirrors the course they teach—and, more important, that students actually read it! Of course, the third edition also benefits from our own experiences using the text, as well as from our experiences teaching mathematics at diverse institutions over the past 30 years.

New to the Third Edition

Exercises

The exercise sets are a major focus of the revision. In response to reviewer and instructor feedback, we've made some significant changes to the exercise sets by rearranging and relabeling exercises, modifying some exercises, and adding many new ones. Of the approximately 10,400 exercises appearing in this edition, 18% are new, and many of the exercises from the second edition were revised for this edition. We analyzed aggregated student usage and performance data from MyLab™ Math for the previous edition of this text. The results of this analysis helped us improve the quality and quantity of exercises that matter the most to instructors and students. We have also simplified the structure of the exercises sets from five parts to the following three:

1. **Getting Started** contains some of the former Review Questions but goes beyond those to include more conceptual exercises, along with new basic skills and short-answer exercises. Our goal in this section is to provide an excellent overall assessment of understanding of the key ideas of a section.
2. **Practice Exercises** consist primarily of exercises from the former Basic Skills, but they also include intermediate-level exercises from the former Further Explorations and Application sections. Unlike previous editions, these exercises are not necessarily organized into groups corresponding to specific examples. For instance, instead of separating out Product Rule exercises from Quotient Rule exercises in Section 3.4, we

have merged these problems into one larger group of exercises. Consequently, specific instructions such as “Use the Product Rule to find the derivative of the following functions” and “Use the Quotient Rule to find the derivative of the given functions” have been replaced with the general instruction “Find the derivative of the following functions.” With Product Rule and Quotient Rule exercises mixed together, students must first choose the correct method for evaluating derivatives before solving the problems.

3. **Explorations and Challenges** consist of more challenging problems and those that extend the content of the section.

We no longer have a section of the exercises called “Applications,” but (somewhat ironically) in eliminating this section, we feel we are providing better coverage of applications because these exercises have been placed strategically *throughout the exercise sets.* Some are in Getting Started, most are in Practice Exercises, and some are in Explorations and Challenges. The applications nearly always have a boldface heading so that the topic of the application is readily apparent.

Regarding the boldface heads that precede exercises: These heads provide instructors with a quick way to discern the topic of a problem when creating assignments. We heard from users of earlier editions, however, that some of these heads provided too much guidance in how to solve a given problem. In this edition, therefore, we eliminated or reworded run-in heads that provided too much information about the solution method for a problem.

Finally, the **Chapter Review exercises** received a major revamp to provide more exercises (particularly intermediate-level problems) and more opportunities for students to choose a strategy of solution. More than 26% of the Chapter Review exercises are new.

Content Changes

Below are noteworthy changes from the previous edition of the text. Many other detailed changes, not noted here, were made to improve the quality of the narrative and exercises. Bullet points with a icon represent major content changes from the previous edition.

Chapter 1 Functions

- Example 2 in Section 1.1 was modified with more emphasis on using algebraic techniques to determine the domain and range of a function. To better illustrate a common feature of limits, we replaced part (c) with a rational function that has a common factor in the numerator and denominator.
- Examples 7 and 8 in Section 1.1 from the second edition (2e) were moved forward in the narrative so that students get an intuitive feel for the composition of two functions using graphs and tables; compositions of functions using algebraic techniques follow.
- Example 10 in Section 1.1, illustrating the importance of secant lines, was made more relevant to students by using real data from a GPS watch during a hike. Corresponding exercises were also added.
- Exercises were added to Section 1.3 to give students practice at finding inverses of functions using the properties of exponential and logarithmic functions.
- New application exercises (investment problems and a biology problem) were added to Section 1.3 to further illustrate the usefulness of logarithmic and exponential functions.

Chapter 2 Limits

- Example 4 in Section 2.2 was revised, emphasizing an algebraic approach to a function with a jump discontinuity, rather than a graphical approach.
- Theorems 2.3 and 2.13 were modified, simplifying the notation to better connect with upcoming material.
- Example 7 in Section 2.3 was added to solidify the notions of left-, right-, and two-sided limits.
- The material explaining the end behavior of exponential and logarithmic functions was reworked, and Example 6 in Section 2.5 was added to show how substitution is used in evaluating limits.
- Exercises were added to Section 2.5 to illustrate the similarities and differences between limits at infinity and infinite limits. We also included some easier exercises in Section 2.5 involving limits at infinity of functions containing square roots.
- Example 5 in Section 2.7 was added to demonstrate an epsilon-delta proof of a limit of a quadratic function.
- We added 17 epsilon-delta exercises to Section 2.7 to provide a greater variety of problems involving limits of quadratic, cubic, trigonometric, and absolute value functions.

Chapter 3 Derivatives

- Chapter 3 now begins with a look back at average and instantaneous velocity, first encountered in Section 2.1, with a corresponding revised example in Section 3.1.
- The derivative at a point and the derivative as a function are now treated separately in Sections 3.1 and 3.2.
- After defining the derivative at a point in Section 3.1 with a supporting example, we added a new subsection: Interpreting the Derivative (with two supporting examples).
- Several exercises were added to Section 3.3 that require students to use the Sum and Constant Rules, together with geometry, to evaluate derivatives.
- The Power Rule for derivatives in Section 3.4 is stated for all real powers (later proved in Section 3.9). Example 4

in Section 3.4 includes two additional parts to highlight this change, and subsequent examples in upcoming sections rely on the more robust version of the Power Rule. The Power Rule for Rational Exponents in Section 3.8 was deleted because of this change.

- We combined the intermediate-level exercises in Section 3.4 involving the Product Rule and Quotient Rule together under one unified set of directions.
- The derivative of e^x still appears early in the chapter, but the derivative of e^{kx} is delayed; it appears only after the Chain Rule is introduced in Section 3.7.
- In Section 3.7, we deleted references to Version 1 and Version 2 of the Chain Rule. Additionally, Chain Rule exercises involving repeated use of the rule were merged with the standard exercises.
- In Section 3.8, we added emphasis on simplifying derivative formulas for implicitly defined functions; see Examples 4 and 5.
- Example 3 in Section 3.11 was replaced; the new version shows how similar triangles are used in solving a related-rates problem.

Chapter 4 Applications of the Derivative

- The Mean Value Theorem (MVT) was moved from Section 4.6 to 4.2 so that the proof of Theorem 4.7 is not delayed. We added exercises to Section 4.2 that help students better understand the MVT geometrically, and we included exercises where the MVT is used to prove some well-known identities and inequalities.
- Example 5 in Section 4.5 was added to give guidance on a certain class of optimization problems.
- Example 3b in Section 4.7 was replaced to better drive home the need to simplify after applying l'Hôpital's Rule.
- Most of the intermediate exercises in Section 4.7 are no longer separated out by the type of indeterminate form, and we added some problems in which l'Hôpital's Rule does not apply.
- Indefinite integrals of trigonometric functions with argument ax (Table 4.9) were relocated to Section 5.5, where they are derived with the Substitution Rule. A similar change was made to Table 4.10.
- Example 7b in Section 4.9 was added to foreshadow a more complete treatment of the domain of an initial value problem found in Chapter 9.
- We added to Section 4.9 a significant number of intermediate antiderivative exercises that require some preliminary work (e.g., factoring, cancellation, expansion) before the antiderivatives can be determined.

Chapter 5 Integration

- Examples 2 and 3 in Section 5.1 on approximating areas were replaced with a friendlier function where the grid points are more transparent; we return to these approximations in Section 5.3, where an exact result is given (Example 3b).
- Three properties of integrals (bounds on definite integrals) were added in Section 5.2 (Table 5.5); the last of these properties is used in the proof of the Fundamental Theorem (Section 5.3).
- Exercises were added to Sections 5.1 and 5.2 where students are required to evaluate Riemann sums using graphs or tables instead of formulas. These exercises will help students better understand the geometric meaning of Riemann sums.
- We added to Section 5.3 more exercises in which the integrand must be simplified before the integrals can be evaluated.
- A proof of Theorem 5.7 is now offered in Section 5.5.
- Table 5.6 lists the general integration formulas that were relocated from Section 4.9 to Section 5.5; Example 4 in Section 5.5 derives these formulas.

Chapter 6 Applications of Integration
Chapter 7 Logarithmic, Exponential, and Hyperbolic Functions

- Chapter 6 from the 2e was split into two chapters in order to match the number of chapters in *Calculus* (Late Transcendentals). The result is a compact Chapter 7.
- Exercises requiring students to evaluate net change using graphs were added to Section 6.1.
- Exercises in Section 6.2 involving area calculations with respect to x and y are now combined under one unified set of directions (so that students must first determine the appropriate variable of integration).
- We increased the number of exercises in Sections 6.3 and 6.4 in which curves are revolved about lines other than the x- and y-axes. We also added introductory exercises that guide students, step by step, through the processes used to find volumes.
- A more gentle introduction to lifting problems (specifically, lifting a chain) was added in Section 6.7 and illustrated in Example 3, accompanied by additional exercises.
- The introduction to exponential growth (Section 7.2) was rewritten to make a clear distinction between the relative growth rate (or percent change) of a quantity and the rate constant k. We revised the narrative so that the equation $y = y_0e^{kt}$ applies to both growth and decay models. This revision resulted in a small change to the half-life formula.
- The variety of applied exercises in Section 7.2 was increased to further illustrate the utility of calculus in the study of exponential growth and decay.

Chapter 8 Integration Techniques

- Table 8.1 now includes four standard trigonometric integrals that previously appeared in the section Trigonometric Integrals (8.3); these integrals are derived in Examples 1 and 2 in Section 8.1.
- A new section (8.6) was added so that students can master integration techniques (that is, choose a strategy) apart from the context given in the previous five sections.
- In Section 8.5 we increased the number and variety of exercises where students must set up the appropriate form of the partial fraction decomposition of a rational function, including more with irreducible quadratic factors.
- A full derivation of Simpson's Rule was added to Section 8.8, accompanied by Example 7, additional figures, and an expanded exercise set.

- The Comparison Test for improper integrals was added to Section 8.9, accompanied by Example 7, a two-part example. New exercises in Section 8.9 include some covering doubly infinite improper integrals over infinite intervals.

Chapter 9 Differential Equations

- The chapter on differential equations that was available only online in the 2e was converted to a chapter of the text, replacing the single-section coverage found in the 2e.
- More attention was given to the domain of an initial value problem, resulting in the addition and revision of several examples and exercises throughout the chapter.

Chapter 10 Sequences and Infinite Series

- The second half of Chapter 10 was reordered: Comparison Tests (Section 10.5), Alternating Series (Section 10.6, which includes the topic of absolute convergence), The Ratio and Root Tests (Section 10.7), and Choosing a Convergence Test (Section 10.8; new section). We split the 2e section that covered the comparison, ratio, and root tests to avoid overwhelming students with too many tests at one time. Section 10.5 focuses entirely on the comparison tests; 39% of the exercises are new. The topic of alternating series now appears before the Ratio and Root Tests so that the latter tests may be stated in their more general form (they now apply to any series rather than only to series with positive terms). The final section (10.8) gives students an opportunity to master convergence tests after encountering each of them separately.
- The terminology associated with sequences (10.2) now includes *bounded above, bounded below,* and *bounded* (rather than only *bounded,* as found in earlier editions).
- Theorem 10.3 (Geometric Sequences) is now developed in the narrative rather than within an example, and an additional example (10.2.3) was added to reinforce the theorem and limit laws from Theorem 10.2.
- Example 5c in Section 10.2 uses mathematical induction to find the limit of a sequence defined recursively; this technique is reinforced in the exercise set.
- Example 3 in Section 10.3 was replaced with telescoping series that are not geometric and that require re-indexing.
- We increased the number and variety of exercises where the student must determine the appropriate series test necessary to determine convergence of a given series.
- We added some easier intermediate-level exercises to Section 10.6, where series are estimated using nth partial sums for a given value of n.
- Properties of Convergent Series (Theorem 10.8) was expanded (two more properties) and moved to Section 10.3 to better balance the material presented in Sections 10.3 and 10.4. Example 4 in Section 10.3 now has two parts to give students more exposure to the theorem.

Chapter 11 Power Series

- Chapter 11 was revised to mesh with the changes made in Chapter 10.
- We included in Section 11.2 more exercises where the student must find the radius and interval of convergence.
- Example 2 in Section 11.3 was added to illustrate how to choose a different center for a series representation of a function when the original series for the function converges to the function on only part of its domain.
- We addressed an issue with the exercises in Section 11.2 of the previous edition by adding more exercises where the intervals of convergence either are closed or contain one, but not both, endpoints.
- We addressed an issue with exercises in the previous edition by adding many exercises that involve power series centered at locations other than 0.

Chapter 12 Parametric and Polar Curves

- The arc length of a two-dimensional curve described by parametric equations was added to Section 12.1, supported by two examples and additional exercises. Area and surfaces of revolution associated with parametric curves were also added to the exercises.
- In Example 3 in Section 12.2, we derive more general polar coordinate equations for circles.
- The arc length of a curve described in polar coordinates is given in Section 12.3.

Chapter 13 Vectors and the Geometry of Space

- The material from the 2e chapter Vectors and Vector-Valued Functions is now covered in this chapter and the following chapter.
- Example 5c in Section 13.1 was added to illustrate how to express a vector as a product of its magnitude and its direction.
- We increased the number of applied vector exercises in Section 13.1, starting with some easier exercises, resulting in a wider gradation of exercises.
- We adopted a more traditional approach to lines and planes; these topics are now covered together in Section 13.5, followed by cylinders and quadric surfaces in Section 13.6. This arrangement gives students early exposure to all the basic three-dimensional objects that they will encounter throughout the remainder of the text.
- A discussion of the distance from a point to a line was moved from the exercises into the narrative, supported with Example 3 in Section 13.5. Example 4 finds the point of intersection of two lines. Several related exercises were added to this section.
- In Section 13.6 there is a larger selection of exercises where the student must identify the quadric surface associated with a given equation. Exercises are also included where students design shapes using quadric surfaces.

Chapter 14 Vector-Valued Functions

- More emphasis was placed on the surface(s) on which a space curve lies in Sections 14.1 and 14.3.

- We added exercises in Section 14.1 where students are asked to find the curve of intersection of two surfaces and where students must verify that a curve lies on a given surface.
- Example 3c in Section 14.3 was added to illustrate how a space curve can be mapped onto a sphere.
- Because the arc length of plane curves (described parametrically in Section 12.1 and with polar coordinates in Section 12.3) was moved to an earlier location in the text, Section 14.4 is now a shorter section.

Chapter 15 Functions of Several Variables

- Equations of planes and quadric surfaces were removed from this chapter and now appear in Chapter 13.
- The notation in Theorem 15.2 was simplified to match changes made to Theorem 2.3.
- Example 7 in Section 15.4 was added to illustrate how the Chain Rule is used to compute second partial derivatives.
- We added more challenging partial derivative exercises to Section 15.3 and more challenging Chain Rule exercises to Section 15.4.
- Example 7 in Section 15.5 was expanded to give students more practice finding equations of curves that lie on surfaces.
- Theorem 15.13 was added in Section 15.5; it's a three-dimensional version of Theorem 15.11.
- Example 7 in Section 15.7 was replaced with a more interesting example; the accompanying figure helps tell the story of maximum/minimum problems and can be used to preview Lagrange multipliers.
- We added to Section 15.7 some basic exercises that help students better understand the second derivative test for functions of two variables.
- Example 1 in Section 15.8 was modified so that using Lagrange multipliers is the clear path to a solution, rather than eliminating one of the variables and using standard techniques. We also make it clear that care must be taken when using the method of Lagrange multipliers on sets that are not closed and bounded (absolute maximum and minimum values may not exist).

Chapter 16 Multiple Integration

- Example 2 in Section 16.3 was modified because it was too similar to Example 1.
- More care was given to the notation used with polar, cylindrical, and spherical coordinates (see, for example, Theorem 16.3 and the development of integration in different coordinate systems).
- Example 3 in Section 16.4 was modified to make the integration a little more transparent and to show that changing variables to polar coordinates is permissible in more than just the xy-plane.
- More multiple integral exercises were added to Sections 16.1, 16.2, and 16.4, where integration by substitution or integration by parts is needed to evaluate the integrals.
- In Section 16.4 we added more exercises in which the integrals must first be evaluated with respect to x or y instead of z. We also included more exercises that require triple integrals to be expressed in several orderings.

Chapter 17 Vector Calculus

- Our approach to scalar line integrals was streamlined; Example 1 in Section 17.2 was modified to reflect this fact.
- We added basic exercises in Section 17.2 emphasizing the geometric meaning of line integrals in a vector field. A subset of exercises was added where line integrals are grouped so that the student must determine the type of line integral before evaluating the integral.
- Theorem 17.5 was added to Section 17.3; it addresses the converse of Theorem 17.4. We also promoted the area of a plane region by a line integral to theorem status (Theorem 17.8 in Section 17.4).
- Example 3 in Section 17.7 was replaced to give an example of a surface whose bounding curve is not a plane curve and to provide an example that buttresses the claims made at the end of the section (that is, Two Final Notes on Stokes' Theorem).
- More line integral exercises were added to Section 17.3 where the student must first find the potential function before evaluating the line integral over a conservative vector field using the Fundamental Theorem of Line Integrals.
- We added to Section 17.7 more challenging surface integrals that are evaluated using Stokes' Theorem.

New to MyLab Math

- **Assignable Exercises** To better support students and instructors, we made the following changes to the assignable exercises:
  - Updated the solution processes in Help Me Solve This and View an Example to better match the techniques used in the text.
  - Added more Setup & Solve exercises to better mirror the types of responses that students are expected to provide on tests. We also added a parallel "standard" version of each Setup & Solve exercise, to allow the instructor to determine which version to assign.
  - Added exercises corresponding to new exercises in the text.

◦ Added exercises where MyLab Math users had identified gaps in coverage in the 2e.

◦ Added extra practice exercises to each section (clearly labeled EXTRA). These "beyond the text" exercises are perfect for chapter reviews, quizzes, and tests.

◦ Analyzed aggregated student usage and performance data from MyLab Math for the previous edition of this text. The results of this analysis helped improve the quality and quantity of exercises that matter the most to instructors and students.

- **Instructional Videos** For each section of the text, there is now a new full-lecture video. Many of these videos make use of Interactive Figures to enhance student understanding of concepts. To make it easier for students to navigate to the specific content they need, each lecture video is segmented into shorter clips (labeled Introduction, Example, or Summary). Both the full lectures and the video segments are assignable within MyLab Math. The videos were created by the following team: Matt Hudelson (Washington State University), Deb Carney and Rebecca Swanson (Colorado School of Mines), Greg Wisloski and Dan Radelet (Indiana University of Pennsylvania), and Nick Ormes (University of Denver).
- **Enhanced Interactive Figures** Incorporating functionality from several standard Interactive Figures makes Enhanced Interactive Figures mathematically richer and ideal for in-class demonstrations. Using a single figure, instructors can illustrate concepts that are difficult for students to visualize and can make important connections to key themes of calculus.
- **Enhanced Sample Assignments** These section-level assignments address gaps in precalculus skills with a personalized review of prerequisites, help keep skills fresh with spaced practice using key calculus concepts, and provide opportunities to work exercises without learning aids so students can check their understanding. They are assignable and editable.
- **Quick Quizzes** have been added to Learning Catalytics™ (an in-class assessment system) for every section of the text.
- **Maple™, Mathematica®, and Texas Instruments® Manuals and Projects** have all been updated to align with the latest software and hardware.

Noteworthy Features

Figures

Given the power of graphics software and the ease with which many students assimilate visual images, we devoted considerable time and deliberation to the figures in this text. Whenever possible, we let the figures communicate essential ideas using annotations reminiscent of an instructor's voice at the board. Readers will quickly find that the figures facilitate learning in new ways.

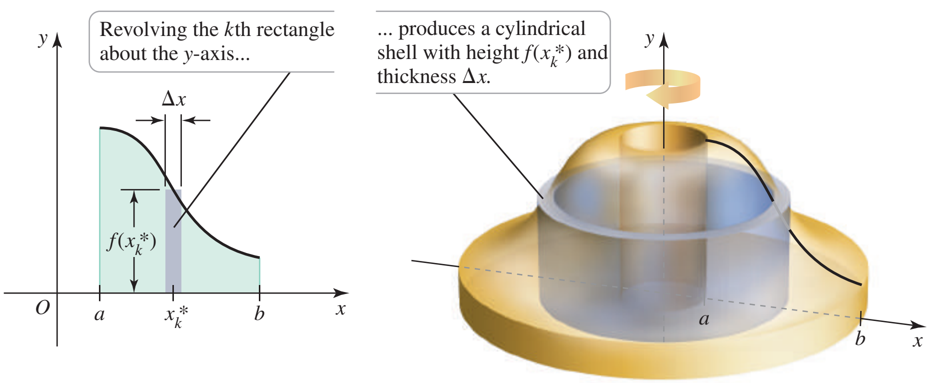

Figure 6.40

Annotated Examples

Worked-out examples feature annotations in blue to guide students through the process of solving the example and to emphasize that each step in a mathematical argument must be rigorously justified. These annotations are designed to echo how instructors "talk through" examples in lecture. They also provide help for students who may struggle with the algebra and trigonometry steps within the solution process.

Quick Checks

The narrative is interspersed with Quick Check questions that encourage students to do the calculus as they are reading about it. These questions resemble the kinds of questions instructors pose in class. Answers to the Quick Check questions are found at the end of the section in which they occur.

Guided Projects

MyLab Math contains 78 Guided Projects that allow students to work in a directed, step-by-step fashion, with various objectives: to carry out extended calculations, to derive physical models, to explore related theoretical topics, or to investigate new applications of calculus. The Guided Projects vividly demonstrate the breadth of calculus and provide a wealth of mathematical excursions that go beyond the typical classroom experience. A list of related Guided Projects is included at the end of each chapter.

Incorporating Technology

We believe that a calculus text should help students strengthen their analytical skills and demonstrate how technology can extend (not replace) those skills. Calculators and graphing utilities are additional tools in the kit, and students must learn when and when not to use them. Our goal is to accommodate the different policies regarding technology adopted by various instructors.

Throughout the text, exercises marked with T indicate that the use of technology—ranging from plotting a function with a graphing calculator to carrying out a calculation using a computer algebra system—may be needed. See page xx for information regarding our technology resource manuals covering Maple, Mathematica, and Texas Instruments graphing calculators.

Text Versions

- **eBook with Interactive Figures** The text is supported by a groundbreaking and award-winning electronic book created by Eric Schulz of Walla Walla Community College. This "live book" runs in Wolfram CDF Player (the free version of Mathematica) and contains the complete text of the print book plus interactive versions of approximately 700 figures. Instructors can use these interactive figures in the classroom to illustrate the important ideas of calculus, and students can explore them while they are reading the text. Our experience confirms that the interactive figures help build students' geometric intuition of calculus. The authors have written Interactive Figure Exercises that can be assigned via MyLab Math so that students can engage with the figures outside of class in a directed way. Available only within MyLab Math, the eBook provides instructors with powerful new teaching tools that expand and enrich the learning experience for students.
- **Other eBook Formats** The text is also available in various stand-alone eBook formats. These are listed in the Pearson online catalog: **www.pearson.com**. MyLab Math also contains an HTML eBook that is screen-reader accessible.
- **Other Print Formats** The text is also available in split editions (Single Variable [Chapters 1–12] and Multivariable [Chapters 10–17]) and in unbound (3-hole punched) formats. Again, see the Pearson online catalog for details: **www.pearson.com.**

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MyLab™ Math is available to accompany Pearson's market-leading text offerings. To give students a consistent tone, voice, and teaching method, each text's flavor and approach are tightly integrated throughout the accompanying MyLab Math course, making learning the material as seamless as possible.

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MyLab Math provides content and tools that help students build a deeper understanding of course content than would otherwise be possible.

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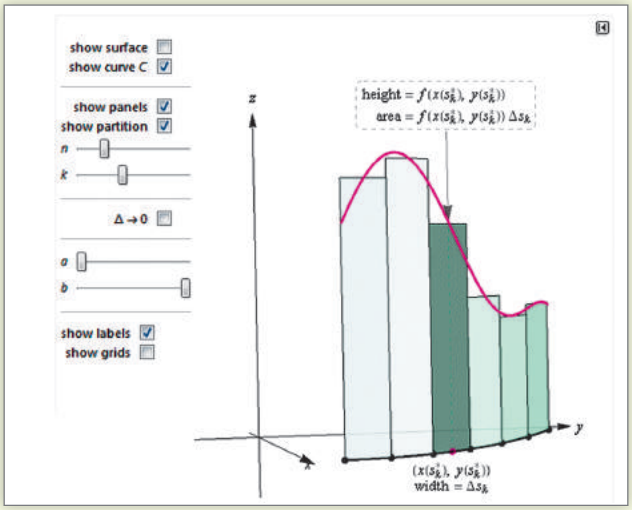
The eBook includes approximately 700 figures that can be manipulated by students to provide a deeper geometric understanding of key concepts and examples as they read and learn new material. Students get unlimited access to the eBook within any MyLab Math course using that edition of the text. The authors have written Interactive Figure Exercises that can be assigned for homework so that students can engage with the figures outside of the classroom.



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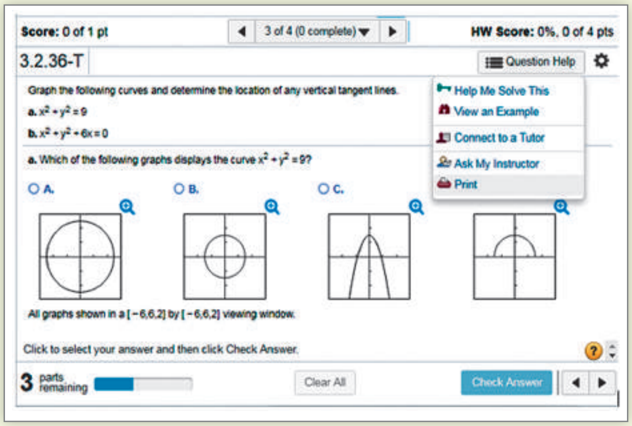


Exercises with Immediate Feedback

The over 8000 homework and practice exercises for this text regenerate algorithmically to give students unlimited opportunity for practice and mastery. MyLab Math provides helpful feedback when students enter incorrect answers and includes the optional learning aids Help Me Solve This, View an Example, videos, and/or the eBook.

NEW! Enhanced Sample Assignments

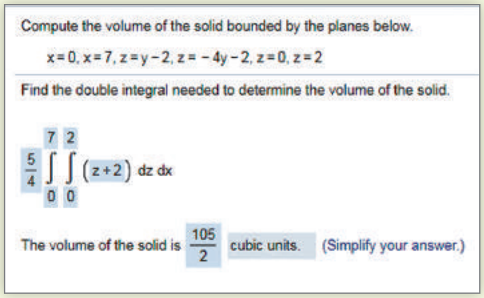
These section-level assignments include just-in-time review of prerequisites, help keep skills fresh with spaced practice of key concepts, and provide opportunities to work exercises without learning aids so students can check their understanding. They are assignable and editable within MyLab Math.

Additional Conceptual Questions

Additional Conceptual Questions focus on deeper, theoretical understanding of the key concepts in calculus. These questions were written by faculty at Cornell University under an NSF grant and are also assignable through Learning Catalytics™.

Setup & Solve Exercises

These exercises require students to show how they set up a problem, as well as the solution, thus better mirroring what is required on tests. This new type of exercise was widely praised by users of the second edition, so more were added to the third edition.



ALL NEW! Instructional Videos

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UPDATED! Technology Manuals (downloadable)

- Maple™ Manual and Projects by Kevin Reeves, East Texas Baptist University
- Mathematica® Manual and Projects by Todd Lee, Elon University
- TI-Graphing Calculator Manual by Elaine McDonald-Newman, Sonoma State University

These manuals cover *Maple* 2017, *Mathematica 11,* and the TI-84 Plus and TI-89, respectively. Each manual provides detailed guidance for integrating the software package or graphing calculator throughout the course, including syntax and commands. The projects include instructions and ready-made application files for Maple and Mathematica. The files can be downloaded from within MyLab Math.

Student's Solutions Manuals (softcover and downloadable)

Single Variable Calculus: Early Transcendentals (Chapters 1–12)
ISBN: 0-13-477048-X | 978-0-13-477048-2
Multivariable Calculus (Chapters 10–17)
ISBN: 0-13-476682-2 | 978-0-13-476682-9
Written by Mark Woodard (Furman University), the Student's Solutions Manual contains worked-out solutions to all the odd-numbered exercises. This manual is available in print and can be downloaded from within MyLab Math.

SUPPORTING INSTRUCTION

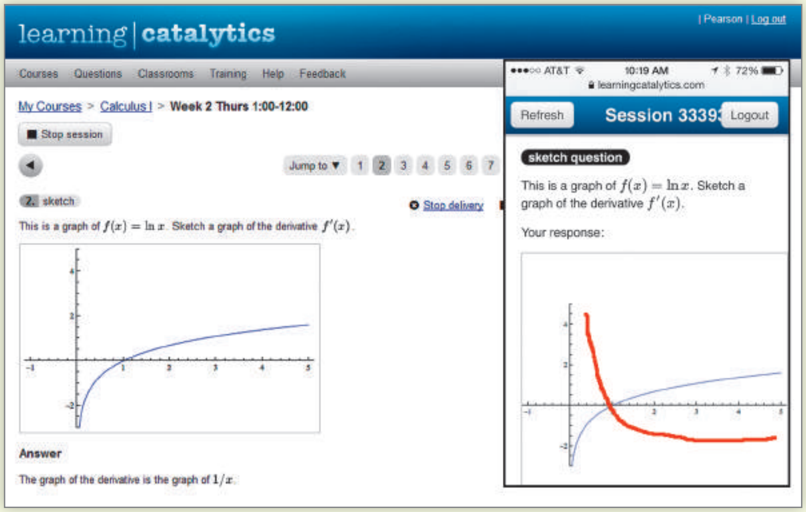
MyLab Math comes from an experienced partner with educational expertise and an eye on the future. It provides resources to help you assess and improve student results at every turn and unparalleled flexibility to create a course tailored to you and your students.

NEW! Enhanced Interactive Figures

Incorporating functionality from several standard Interactive Figures makes Enhanced Interactive Figures mathematically richer and ideal for in-class demonstrations. Using a single enhanced figure, instructors can illustrate concepts that are difficult for students to visualize and can make important connections to key themes of calculus.

Learning Catalytics

Now included in all MyLab Math courses, this student response tool uses students' smartphones, tablets, or laptops to engage them in more interactive tasks and thinking during lecture. Learning Catalytics™ fosters student engagement and peer-to-peer learning with real-time analytics. Access pre-built exercises created specifically for calculus, including Quick Quiz exercises for each section of the text.



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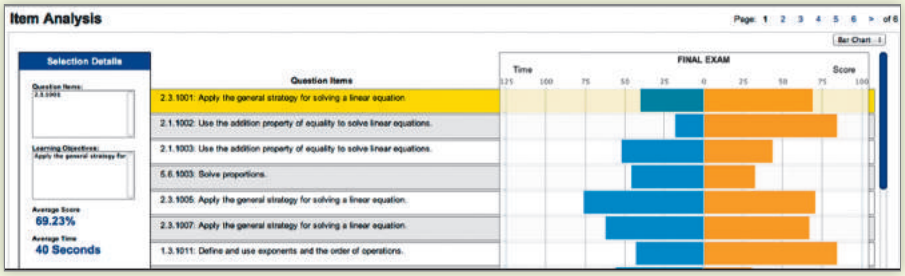
PowerPoint Lecture Resources (downloadable)

Slides contain presentation resources such as key concepts, examples, definitions, figures, and tables from this text. They can be downloaded from within MyLab Math or from Pearson's online catalog at **www.pearson.com**.

Comprehensive Gradebook

The gradebook includes enhanced reporting functionality, such as item analysis and a reporting dashboard to enable you to efficiently manage your course. Student performance data are presented at the class, section, and program levels in an accessible, visual manner so you'll have the information you need to keep your students on track.



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TestGen® (**www.pearson.com/testgen**) enables instructors to build, edit, print, and administer tests using a computerized bank of questions developed to cover all the objectives of the text. TestGen is algorithmically based, allowing instructors to create multiple, but equivalent, versions of the same question or test with the click of a button. Instructors can also modify test bank questions and/or add new questions. The software and test bank are available for download from Pearson's online catalog, **www.pearson.com**. The questions are also assignable in MyLab Math.

Instructor's Solutions Manual (downloadable)

Written by Mark Woodard (Furman University), the Instructor's Solutions Manual contains complete solutions to all the exercises in the text. It can be downloaded from within MyLab Math or from Pearson's online catalog, **www.pearson.com**.

Instructor's Resource Guide (downloadable)

This resource includes Guided Projects that require students to make connections between concepts and applications from different parts of the calculus course. They are correlated to specific chapters of the text and can be assigned as individual or group work. The files can be downloaded from within MyLab Math or from Pearson's online catalog, **www.pearson.com**.

Accessibility

Pearson works continuously to ensure that our products are as accessible as possible to all students. We are working toward achieving WCAG 2.0 Level AA and Section 508 standards, as expressed in the Pearson Guidelines for Accessible Educational Web Media, **www.pearson.com/mylab/math/accessibility**.

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Credits

Chapter opener art: Andrey Pavlov/123RF

Chapter 1

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Chapter 3

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1

Functions

Chapter Preview Mathematics is a language with an alphabet, a vocabulary, and many rules. Before beginning your calculus journey, you should be familiar with the elements of this language. Among these elements are algebra skills; the notation and terminology for various sets of real numbers; and the descriptions of lines, circles, and other basic sets in the coordinate plane. A review of this material is found in Appendix B, online at goo.gl/6DCbbM. This chapter begins with the fundamental concept of a function and then presents the entire cast of functions needed for calculus: polynomials, rational functions, algebraic functions, exponential and logarithmic functions, and the trigonometric functions, along with their inverses. Before you begin studying calculus, it is important that you master the ideas in this chapter.

1.1 Review of Functions

Everywhere around us we see relationships among quantities, or **variables**. For example, the consumer price index changes in time and the temperature of the ocean varies with latitude. These relationships can often be expressed by mathematical objects called *functions*. Calculus is the study of functions, and because we use functions to describe the world around us, calculus is a universal language for human inquiry.

> **DEFINITION Function**
>
> A **function** f is a rule that assigns to each value x in a set D a *unique* value denoted $f(x)$. The set D is the **domain** of the function. The **range** is the set of all values of $f(x)$ produced as x varies over the entire domain (Figure 1.1).

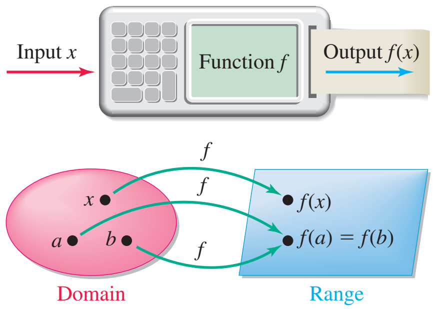

Figure 1.1

The **independent variable** is the variable associated with the domain; the **dependent variable** belongs to the range. The **graph** of a function f is the set of all points (x, y) in the xy-plane that satisfy the equation $y = f(x)$. The **argument** of a function is the expression on which the function works. For example, x is the argument when we write $f(x)$. Similarly, 2 is the argument in $f(2)$ and $x^2 + 4$ is the argument in $f(x^2 + 4)$.

QUICK CHECK 1 If $f(x) = x^2 - 2x$, find $f(-1)$, $f(x^2)$, $f(t)$, and $f(p - 1)$. ◄

The requirement that a function assigns a *unique* value of the dependent variable to each value in the domain is expressed in the vertical line test (Figure 1.2a). For example, the outside temperature as it varies over the course of a day is a function of time (Figure 1.2b).

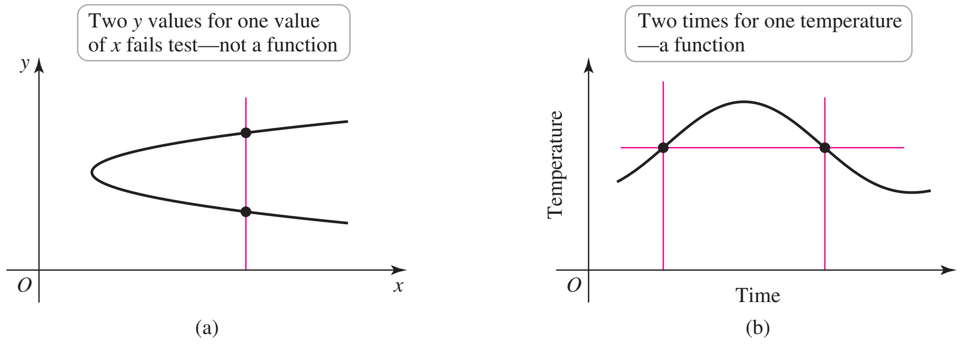

Figure 1.2

➤ If the domain is not specified, we take it to be the set of all values of x for which f is defined. We will see shortly that the domain and range of a function may be restricted by the context of the problem.

➤ A set of points or a graph that does *not* correspond to a function represents a **relation** between the variables. All functions are relations, but not all relations are functions.

Vertical Line Test

A graph represents a function if and only if it passes the **vertical line test**: Every vertical line intersects the graph at most once. A graph that fails this test does not represent a function.

EXAMPLE 1 **Identifying functions** State whether each graph in Figure 1.3 represents a function.

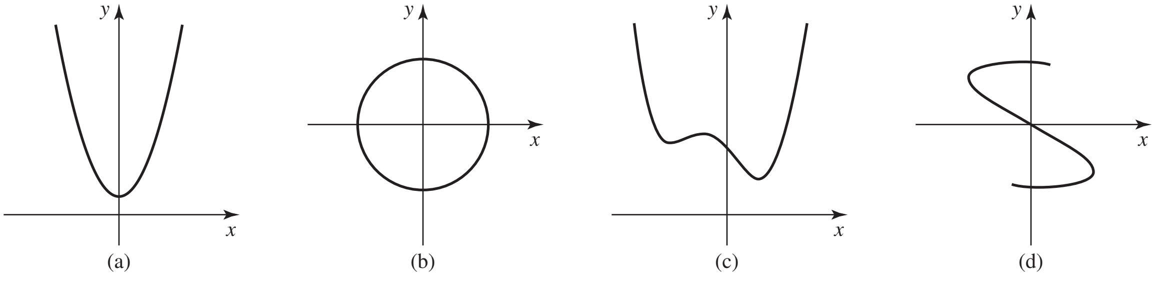

Figure 1.3

SOLUTION The vertical line test indicates that only graphs (a) and (c) represent functions. In graphs (b) and (d), there are vertical lines that intersect the graph more than once. Equivalently, there are values of x that correspond to more than one value of y. Therefore, graphs (b) and (d) do not pass the vertical line test and do not represent functions.

Related Exercise 3 ◄

EXAMPLE 2 **Domain and range** Determine the domain and range of each function.

a. $f(x) = x^2 + 1$ **b.** $g(x) = \sqrt{4 - x^2}$ **c.** $h(x) = \dfrac{x^2 - 3x + 2}{x - 1}$

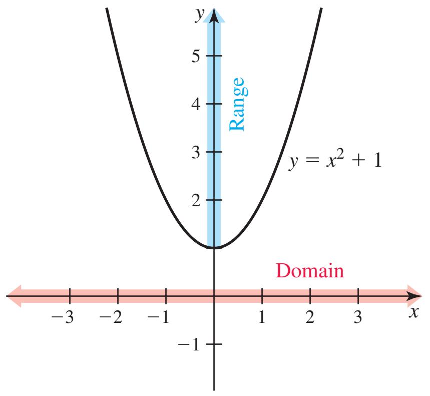

Figure 1.4

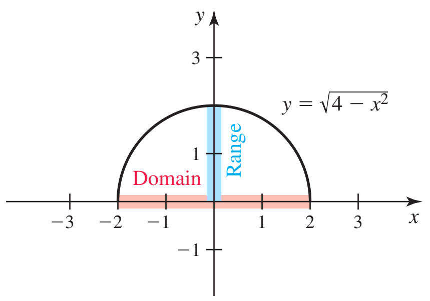

Figure 1.5

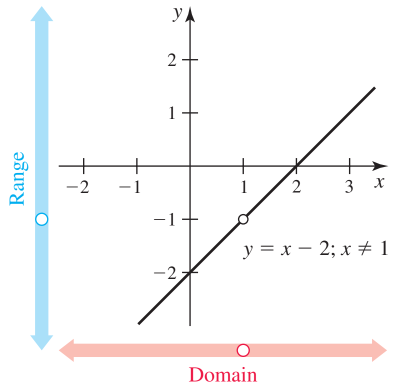

Figure 1.6

SOLUTION

a. Note that f is defined for all values of x; therefore, its domain is the set of all real numbers, written $(-\infty, \infty)$ or $\mathbb{R}$. Because $x^2 \geq 0$ for all x, it follows that $x^2 + 1 \geq 1$, which implies that the range of f is $[1, \infty)$. **Figure 1.4** shows the graph of f along with its domain and range.

b. Functions involving square roots are defined provided the quantity under the root is nonnegative (additional restrictions may also apply). In this case, the function g is defined provided $4 - x^2 \geq 0$, which means $x^2 \leq 4$, or $-2 \leq x \leq 2$. Therefore, the domain of g is $[-2, 2]$. The graph of $g(x) = \sqrt{4 - x^2}$ is the upper half of a circle centered at the origin with radius 2 (**Figure 1.5**; see Appendix B, online at goo.gl/6DCbbM). From the graph we see that the range of g is $[0, 2]$.

c. The function h is defined for all values of $x \neq 1$, so its domain is $\{x: x \neq 1\}$. Factoring the numerator, we find that

$$h(x) = \frac{x^2 - 3x + 2}{x - 1} = \frac{(x-1)(x-2)}{x - 1} = x - 2, \text{ provided } x \neq 1.$$

The graph of $y = h(x)$, shown in **Figure 1.6**, is identical to the graph of the line $y = x - 2$ except that it has a hole at $(1, -1)$ because h is undefined at $x = 1$. Therefore, the range of h is $\{y: y \neq -1\}$. *Related Exercises 23, 25*

EXAMPLE 3 **Domain and range in context** At time $t = 0$, a stone is thrown vertically upward from the ground at a speed of 30 m/s. Its height h above the ground in meters (neglecting air resistance) is approximated by the function $f(t) = 30t - 5t^2$, where t is measured in seconds. Find the domain and range of f in the context of this particular problem.

SOLUTION Although f is defined for all values of t, the only relevant times are between the time the stone is thrown $(t = 0)$ and the time it strikes the ground, when $h = 0$. Solving the equation $h = 30t - 5t^2 = 0$, we find that

$$\begin{aligned} 30t - 5t^2 &= 0 \\ 5t(6 - t) &= 0 && \text{Factor.} \\ 5t = 0 \quad \text{or} \quad 6 - t &= 0 && \text{Set each factor equal to 0.} \\ t = 0 \quad \text{or} \quad t &= 6. && \text{Solve.} \end{aligned}$$

Therefore, the stone leaves the ground at $t = 0$ and returns to the ground at $t = 6$. An appropriate domain that fits the context of this problem is $\{t: 0 \leq t \leq 6\}$. The range consists of all values of $h = 30t - 5t^2$ as t varies over $[0, 6]$. The largest value of h occurs when the stone reaches its highest point at $t = 3$ (halfway through its flight), which is $h = f(3) = 45$. Therefore, the range is $[0, 45]$. These observations are confirmed by the graph of the height function (**Figure 1.7**). Note that this graph is *not* the trajectory of the stone; the stone moves vertically.

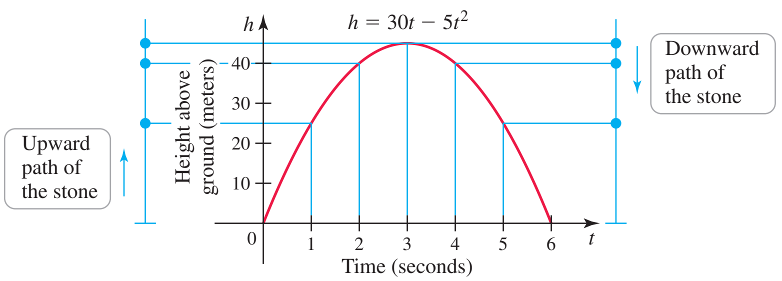

Figure 1.7 *Related Exercises 8–9*

QUICK CHECK 2 State the domain and range of $f(x) = (x^2 + 1)^{-1}$.

Composite Functions

Functions may be combined using sums $(f + g)$, differences $(f - g)$, products (fg), or quotients (f/g). The process called *composition* also produces new functions.

➤ In the composition $y = f(g(x))$, f is the *outer function* and g is the *inner function*.

DEFINITION Composite Functions

Given two functions f and g, the composite function $f \circ g$ is defined by $(f \circ g)(x) = f(g(x))$. It is evaluated in two steps: $y = f(u)$, where $u = g(x)$. The domain of $f \circ g$ consists of all x in the domain of g such that $u = g(x)$ is in the domain of f (**Figure 1.8**).

➤ Three different notations for intervals on the real number line will be used throughout the text:
- $[-2, 3)$ is an example of interval notation,
- $-2 \le x < 3$ is inequality notation, and
- $\{x: -2 \le x < 3\}$ is set notation.

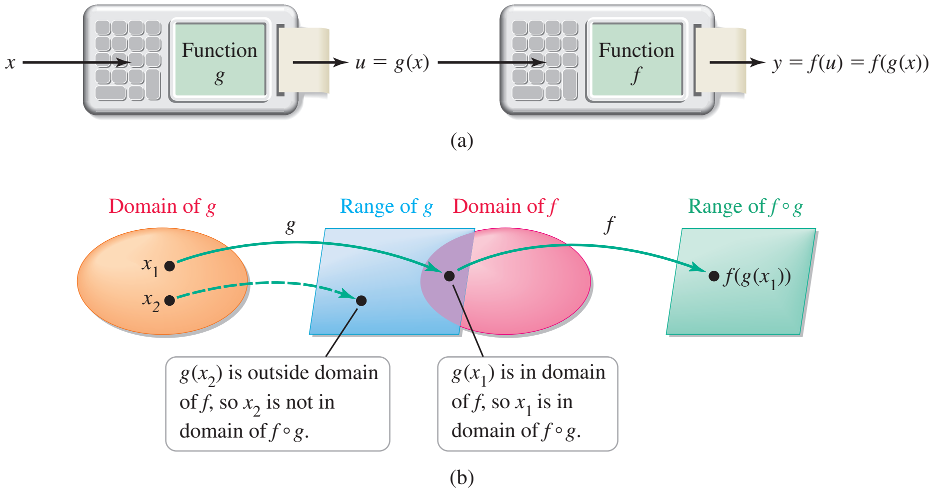

Figure 1.8

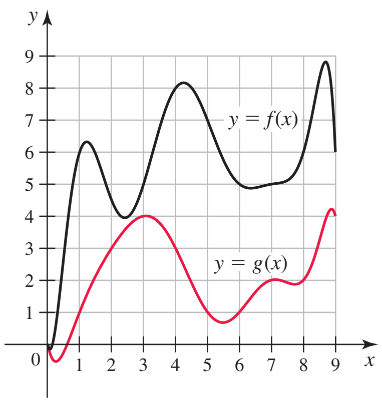

Figure 1.9

EXAMPLE 4 Using graphs to evaluate composite functions Use the graphs of f and g in **Figure 1.9** to find the following values.

a. $f(g(3))$ **b.** $g(f(3))$ **c.** $f(f(4))$ **d.** $f(g(f(8)))$

SOLUTION

a. The graphs indicate that $g(3) = 4$ and $f(4) = 8$, so $f(g(3)) = f(4) = 8$.

b. We see that $g(f(3)) = g(5) = 1$. Observe that $f(g(3)) \neq g(f(3))$.

c. In this case, $f(\underbrace{f(4)}_{8}) = f(8) = 6$.

d. Starting on the inside,

$$f(g(\underbrace{f(8)}_{6})) = f(\underbrace{g(6)}_{1}) = f(1) = 6.$$

Related Exercise 15 ◄

EXAMPLE 5 Using a table to evaluate composite functions Use the function values in the table to evaluate the following composite functions.

a. $(f \circ g)(0)$ **b.** $g(f(-1))$ **c.** $f(g(g(-1)))$

x	-2	-1	0	1	2
$f(x)$	0	1	3	4	2
$g(x)$	-1	0	-2	-3	-4

SOLUTION

a. Using the table, we see that $g(0) = -2$ and $f(-2) = 0$. Therefore, $(f \circ g)(0) = 0$.

b. Because $f(-1) = 1$ and $g(1) = -3$, it follows that $g(f(-1)) = -3$.

c. Starting with the inner function,

$$f(g(\underbrace{g(-1)}_{0})) = f(\underbrace{g(0)}_{-2}) = f(-2) = 0.$$

Related Exercise 16 ◄

EXAMPLE 6 **Composite functions and notation** Let $f(x) = 3x^2 - x$ and $g(x) = 1/x$. Simplify the following expressions.

a. $f(5p + 1)$ **b.** $g(1/x)$ **c.** $f(g(x))$ **d.** $g(f(x))$

SOLUTION In each case, the functions work on their arguments.

a. The argument of f is $5p + 1$, so

$$f(5p + 1) = 3(5p + 1)^2 - (5p + 1) = 75p^2 + 25p + 2.$$

b. Because g requires taking the reciprocal of the argument, we take the reciprocal of $1/x$ and find that $g(1/x) = 1/(1/x) = x$.

c. The argument of f is $g(x)$, so

$$f(g(x)) = f\left(\frac{1}{x}\right) = 3\left(\frac{1}{x}\right)^2 - \left(\frac{1}{x}\right) = \frac{3}{x^2} - \frac{1}{x} = \frac{3 - x}{x^2}.$$

➤ Examples 6c and 6d demonstrate that, in general,

$$f(g(x)) \neq g(f(x)).$$

d. The argument of g is $f(x)$, so

$$g(f(x)) = g(3x^2 - x) = \frac{1}{3x^2 - x}.$$

Related Exercises 33–37 ◀

EXAMPLE 7 **Working with composite functions** Identify possible choices for the inner and outer functions in the following composite functions. Give the domain of the composite function.

a. $h(x) = \sqrt{9x - x^2}$ **b.** $h(x) = \dfrac{2}{(x^2 - 1)^3}$

SOLUTION

a. An obvious outer function is $f(x) = \sqrt{x}$, which works on the inner function $g(x) = 9x - x^2$. Therefore, h can be expressed as $h = f \circ g$ or $h(x) = f(g(x))$. The domain of $f \circ g$ consists of all values of x such that $9x - x^2 \geq 0$. Solving this inequality gives $\{x: 0 \leq x \leq 9\}$ as the domain of $f \circ g$.

➤ Techniques for solving inequalities are discussed in Appendix B, online at goo.gl/6DCbbM.

b. A good choice for an outer function is $f(x) = 2/x^3 = 2x^{-3}$, which works on the inner function $g(x) = x^2 - 1$. Therefore, h can be expressed as $h = f \circ g$ or $h(x) = f(g(x))$. The domain of $f \circ g$ consists of all values of $g(x)$ such that $g(x) \neq 0$, which is $\{x: x \neq \pm 1\}$.

Related Exercises 44–45 ◀

EXAMPLE 8 **More composite functions** Given $f(x) = \sqrt[3]{x}$ and $g(x) = x^2 - x - 6$, find the following composite functions and their domains.

a. $g \circ f$ **b.** $g \circ g$

SOLUTION

a. We have

$$(g \circ f)(x) = g(f(x)) = g(\sqrt[3]{x}) = \underbrace{(\sqrt[3]{x})}_{f(x)}{}^2 - \underbrace{\sqrt[3]{x}}_{f(x)} - 6 = x^{2/3} - x^{1/3} - 6.$$

Because the domains of f and g are $(-\infty, \infty)$, the domain of $f \circ g$ is also $(-\infty, \infty)$.

b. In this case, we have the composition of two polynomials:

$$\begin{aligned}(g \circ g)(x) &= g(g(x)) \\ &= g(x^2 - x - 6) \\ &= \underbrace{(x^2 - x - 6)}_{g(x)}{}^2 - \underbrace{(x^2 - x - 6)}_{g(x)} - 6 \\ &= x^4 - 2x^3 - 12x^2 + 13x + 36.\end{aligned}$$

The domain of the composition of two polynomials is $(-\infty, \infty)$.

Related Exercises 47–48 ◀

QUICK CHECK 3 If $f(x) = x^2 + 1$ and $g(x) = x^2$, find $f \circ g$ and $g \circ f$. ◀

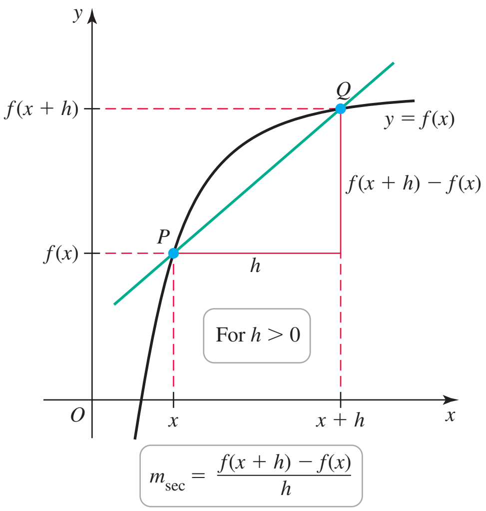

Figure 1.10

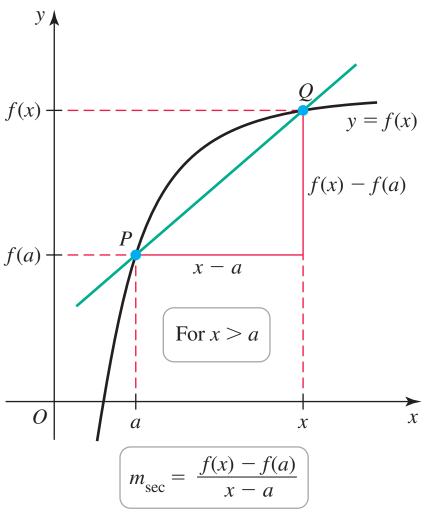

Figure 1.11

➤ Treat $f(x + h)$ like the composition $f(g(x))$, where $x + h$ plays the role of $g(x)$. It may help to establish a pattern in your mind before evaluating $f(x + h)$. For instance, using the function in Example 9a, we have

$$f(x) = 3x^2 - x;$$
$$f(12) = 3 \cdot 12^2 - 12;$$
$$f(b) = 3b^2 - b;$$
$$f(\text{math}) = 3 \cdot \text{math}^2 - \text{math};$$

therefore,

$$f(x + h) = 3(x + h)^2 - (x + h).$$

➤ See the front papers of this text for a review of factoring formulas.

Secant Lines and the Difference Quotient

As you will see shortly, slopes of lines and curves play a fundamental role in calculus. **Figure 1.10** shows two points $P(x, f(x))$ and $Q(x + h, f(x + h))$ on the graph of $y = f(x)$ in the case that $h > 0$. A line through any two points on a curve is called a **secant line**; its importance in the study of calculus is explained in Chapters 2 and 3. For now, we focus on the slope of the secant line through P and Q, which is denoted m_{sec} and is given by

$$m_{\text{sec}} = \frac{\text{change in } y}{\text{change in } x} = \frac{f(x + h) - f(x)}{(x + h) - x} = \frac{f(x + h) - f(x)}{h}.$$

The slope formula $\dfrac{f(x + h) - f(x)}{h}$ is also known as a **difference quotient**, and it can be expressed in several ways depending on how the coordinates of P and Q are labeled. For example, given the coordinates $P(a, f(a))$ and $Q(x, f(x))$ (**Figure 1.11**), the difference quotient is

$$m_{\text{sec}} = \frac{f(x) - f(a)}{x - a}.$$

We interpret the slope of the secant line in this form as the **average rate of change** of f over the interval $[a, x]$.

EXAMPLE 9 **Working with difference quotients**

a. Simplify the difference quotient $\dfrac{f(x + h) - f(x)}{h}$, for $f(x) = 3x^2 - x$.

b. Simplify the difference quotient $\dfrac{f(x) - f(a)}{x - a}$, for $f(x) = x^3$.

SOLUTION

a. First note that $f(x + h) = 3(x + h)^2 - (x + h)$. We substitute this expression into the difference quotient and simplify:

$$\begin{aligned}
\frac{f(x + h) - f(x)}{h} &= \frac{\overbrace{3(x + h)^2 - (x + h)}^{f(x+h)} - \overbrace{(3x^2 - x)}^{f(x)}}{h} \\
&= \frac{3(x^2 + 2xh + h^2) - (x + h) - (3x^2 - x)}{h} && \text{Expand } (x + h)^2. \\
&= \frac{3x^2 + 6xh + 3h^2 - x - h - 3x^2 + x}{h} && \text{Distribute.} \\
&= \frac{6xh + 3h^2 - h}{h} && \text{Simplify.} \\
&= \frac{h(6x + 3h - 1)}{h} = 6x + 3h - 1. && \text{Factor and simplify.}
\end{aligned}$$

b. The factoring formula for the difference of perfect cubes is needed:

$$\begin{aligned}
\frac{f(x) - f(a)}{x - a} &= \frac{x^3 - a^3}{x - a} \\
&= \frac{(x - a)(x^2 + ax + a^2)}{x - a} && \text{Factoring formula} \\
&= x^2 + ax + a^2. && \text{Simplify.}
\end{aligned}$$

Related Exercises 66, 72 ◄

EXAMPLE 10 **Interpreting the slope of the secant line** The position of a hiker on a trail at various times t is recorded by a GPS watch worn by the hiker. These data are then uploaded to a computer to produce the graph of the distance function $d = f(t)$ shown in Figure 1.12, where d measures the distance traveled on the trail in miles and t is the elapsed time in hours from the beginning of the hike.

a. Find the slope of the secant line that passes through the points on the graph corresponding to the trail segment between milepost 3 and milepost 5, and interpret the result.

b. Estimate the slope of the secant line that passes through points A and B in Figure 1.12, and compare it to the slope of the secant line found in part (a).

➤ Figure 1.12 contains actual GPS data collected in Rocky Mountain National Park. See Exercises 75–76 for another look at the data set.

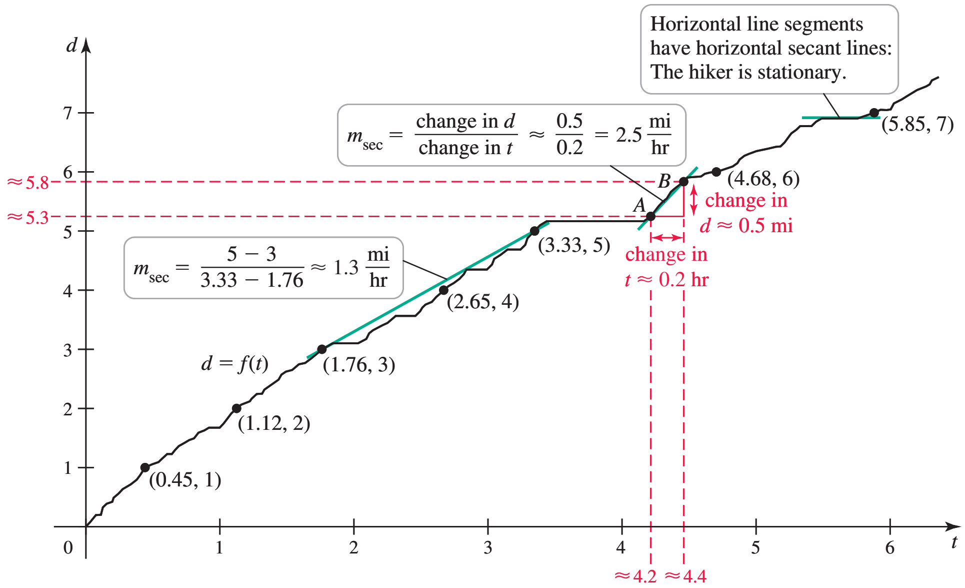

Figure 1.12

SOLUTION

a. We see from the graph of $d = f(t)$ that 1.76 hours (about 1 hour and 46 minutes) has elapsed when the hiker arrives at milepost 3, while milepost 5 is reached 3.33 hours into the hike. This information is also expressed as $f(1.76) = 3$ and $f(3.33) = 5$. To find the slope of the secant line through these points, we compute the change in distance divided by the change in time:

$$m_{sec} = \frac{f(3.33) - f(1.76)}{3.33 - 1.76} = \frac{5 - 3}{3.33 - 1.76} \approx 1.3 \frac{\text{mi}}{\text{hr}}.$$

The units provide a clue about the physical meaning of the slope: It measures the average rate at which the distance changes per hour, which is the average speed of the hiker. In this case, the hiker walks with an average speed of approximately 1.3 mi/hr between mileposts 3 and 5.

b. From the graph we see that the coordinates of points A and B are approximately $(4.2, 5.3)$ and $(4.4, 5.8)$, respectively, which implies the hiker walks $5.8 - 5.3 = 0.5$ mi in $4.4 - 4.2 = 0.2$ hr. The slope of the secant line through A and B is

$$m_{sec} = \frac{\text{change in } d}{\text{change in } t} \approx \frac{0.5}{0.2} = 2.5 \frac{\text{mi}}{\text{hr}}.$$

For this segment of the trail, the hiker walks at an average speed of about 2.5 mi/hr, nearly twice as fast as the average speed computed in part (a). Expressed another way, steep sections of the distance curve yield steep secant lines, which correspond to faster average hiking speeds. Conversely, any secant line with slope equal to 0 corresponds to an average speed of 0. Looking one last time at Figure 1.12, we can identify the time intervals during which the hiker was resting alongside the trail—whenever the distance curve is horizontal, the hiker is not moving.

Related Exercise 75 ➤

QUICK CHECK 4 Refer to Figure 1.12. Find the hiker's average speed during the first mile of the trail and then determine the hiker's average speed in the time interval from 3.9 to 4.1 hours. ➤

Symmetry

The word *symmetry* has many meanings in mathematics. Here we consider symmetries of graphs and the relations they represent. Taking advantage of symmetry often saves time and leads to insights.

DEFINITION Symmetry in Graphs

A graph is **symmetric with respect to the y-axis** if whenever the point (x, y) is on the graph, the point $(-x, y)$ is also on the graph. This property means that the graph is unchanged when reflected across the y-axis (**Figure 1.13a**).

A graph is **symmetric with respect to the x-axis** if whenever the point (x, y) is on the graph, the point $(x, -y)$ is also on the graph. This property means that the graph is unchanged when reflected across the x-axis (**Figure 1.13b**).

A graph is **symmetric with respect to the origin** if whenever the point (x, y) is on the graph, the point $(-x, -y)$ is also on the graph (**Figure 1.13c**). Symmetry about both the x- and y-axes implies symmetry about the origin, but not vice versa.

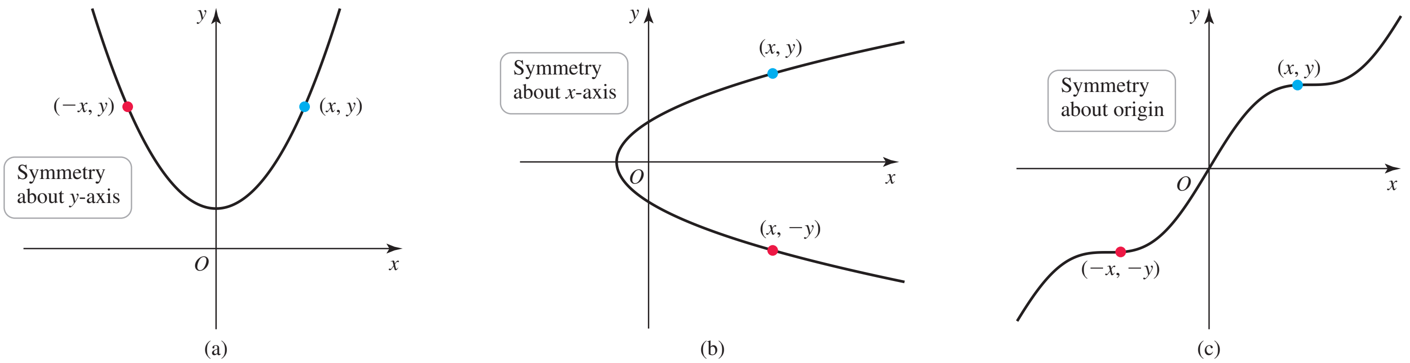

Figure 1.13

DEFINITION Symmetry in Functions

An **even function** f has the property that $f(-x) = f(x)$, for all x in the domain. The graph of an even function is symmetric about the y-axis.

An **odd function** f has the property that $f(-x) = -f(x)$, for all x in the domain. The graph of an odd function is symmetric about the origin.

Polynomials consisting of only even powers of the variable (of the form x^{2n}, where n is a nonnegative integer) are even functions. Polynomials consisting of only odd powers of the variable (of the form x^{2n+1}, where n is a nonnegative integer) are odd functions.

QUICK CHECK 5 Explain why the graph of a nonzero function is never symmetric with respect to the x-axis. ➤

EXAMPLE 11 **Identifying symmetry in functions** Identify the symmetry, if any, in the following functions.

a. $f(x) = x^4 - 2x^2 - 20$ **b.** $g(x) = x^3 - 3x + 1$ **c.** $h(x) = \dfrac{1}{x^3 - x}$

SOLUTION

a. The function f consists of only even powers of x (where $20 = 20 \cdot 1 = 20x^0$ and x^0 is considered an even power). Therefore, f is an even function (Figure 1.14). This fact is verified by showing that $f(-x) = f(x)$:

$$f(-x) = (-x)^4 - 2(-x)^2 - 20 = x^4 - 2x^2 - 20 = f(x).$$

b. The function g consists of two odd powers and one even power (again, $1 = x^0$ is an even power). Therefore, we expect that g has no symmetry about the y-axis or the origin (Figure 1.15). Note that

$$g(-x) = (-x)^3 - 3(-x) + 1 = -x^3 + 3x + 1,$$

so $g(-x)$ equals neither $g(x)$ nor $-g(x)$; therefore, g has no symmetry.

c. In this case, h is a composition of an odd function $f(x) = 1/x$ with an odd function $g(x) = x^3 - x$. Note that

$$h(-x) = \frac{1}{(-x)^3 - (-x)} = -\frac{1}{x^3 - x} = -h(x).$$

Because $h(-x) = -h(x)$, h is an odd function (Figure 1.16).

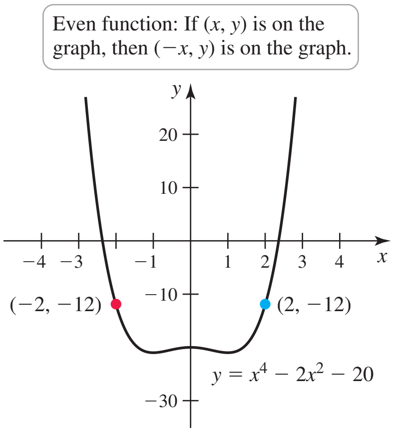

Figure 1.14

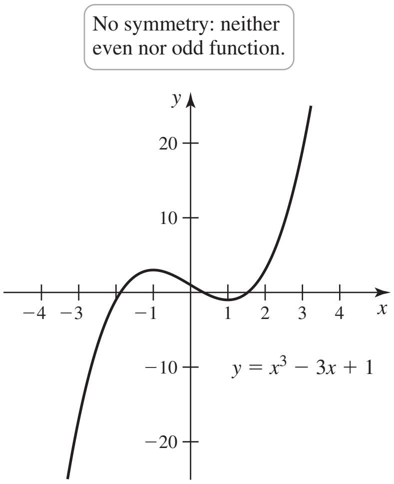

Figure 1.15

➤ The symmetry of compositions of even and odd functions is considered in Exercises 101–104.

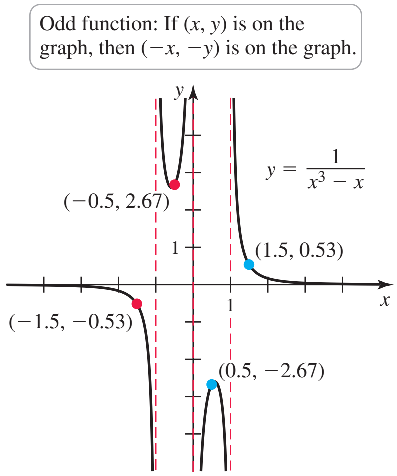

Figure 1.16

Related Exercises 79–81 ➤

SECTION 1.1 EXERCISES

Getting Started

1. Use the terms *domain*, *range*, *independent variable*, and *dependent variable* to explain how a function relates one variable to another variable.

2. Is the independent variable of a function associated with the domain or range? Is the dependent variable associated with the domain or range?

3. Decide whether graph A, graph B, or both represent functions.